Precalculus: Are-you-ready Test

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Find the answer of each problems in a **well-organized step-by-step** *manner. The required background includes (1) the Pythagorean theorem, (2) arithmetic and geometric sequences, (3) factorization of algebraic expressions, (4) quadratic equations, (5) analysis of simple function behavior, (6) circles in the Cartesian coordinate plane, (7) complex numbers, and (8) angles.*

Problem 1

(1) Find the hypotenuse of a right triangle with legs of length 1 and $\sqrt{3}$?
(2) If a right triangle has the hypotenuse of length 82 and a leg of length 80, find the length of the other leg.
(3) If a right triangle has two legs of the same length, find the three angles of the right triangle.

Problem 2

A list of 7 consecutive even numbers has the arithmetic mean/average equal to 16.
(1) What is the smallest even number in the list?
(2) What is the largest even number in the list?
(3) What is the sum of these 7 consecutive even numbers?

Problem 3

The people get infected by a contagious disease *d*ouble every week. In the first week, there is only 10 infected people.
(1) How may people will be infected in the eighth week?
(2) In which week will there be 5120 infected people?

Problem 4

Solve each equation below. If the equation has no solution, write down no solutions.
(1) $3x - 5 - (2x + 4) - 2(x - 2) = -3 + 5x$
(2) $2x^2 - 4x + 1 = 0$
(3) $2x + 3y = 13$, $4x - 5y = -7$
(4) $x + y = 2$, $2x + 2y = 3$
(5) $\sqrt{x+1} + 2/\sqrt{x+1} = 3$

Problem 5

Factor each expression (as product of factors) as much as possible.
(1) $2x^2y - 8y$
(2) $x^2 - 3x + 2$
(3) $2x^2 + x - 1$

Problem 6

In a coordinate plane, a circle with center $(5, 2)$ passes through the point $(-7, 7)$.

(1) What is the radius of the circle?

(2) What is the circumference and area of the circle?

(3) What is the equation of the circle?

(4) Does the circle pass through the point $(17, 7)$?

(5) Find the points of intersection of the circle with the line $y = x + 14$.

Problem 7

$$y = \frac{2}{x - 1}$$

(1) x can not take which value?

(2) If $x = 5$, what is the value of y?

(3) If x is larger and larger and approaches the infinity, what value does y approach?

(4) If $y = 1$, what is the value of x?

(5) If $x > 1$ and x approach 1 (closer and closer to 1), what does y approach?

Problem 8

Find the value of each of the following (where $i = \sqrt{-1}$).

(1) $4(3 + 4i) - 3(-2 - 9i)$

(2) $(2 + i)(2 - i)$

(3) $(3 + 2i)/(2 - i)$

(4) $|6 - 8i|$

Problem 9

(1) Point A, B and C lie on a circle in that order such that the measure of arc AB is 110° and the meansure of arc BC is 120°. What is the measure in degrees of $\angle ABC$?

(2) What is the measure of the angle in degrees formed by the minute and hour hands of a clock at 2:30?