

§2.1-2.5

Functions and Graphs of Functions

What is a mathematical function?

- * A function : A correspondence / relation / mapping that assigns **ONE** output to each input according to a rule.

Notations :

$$f(x) = 2x + 3$$

name of
the function

variable for
the input

rule to produce
the output

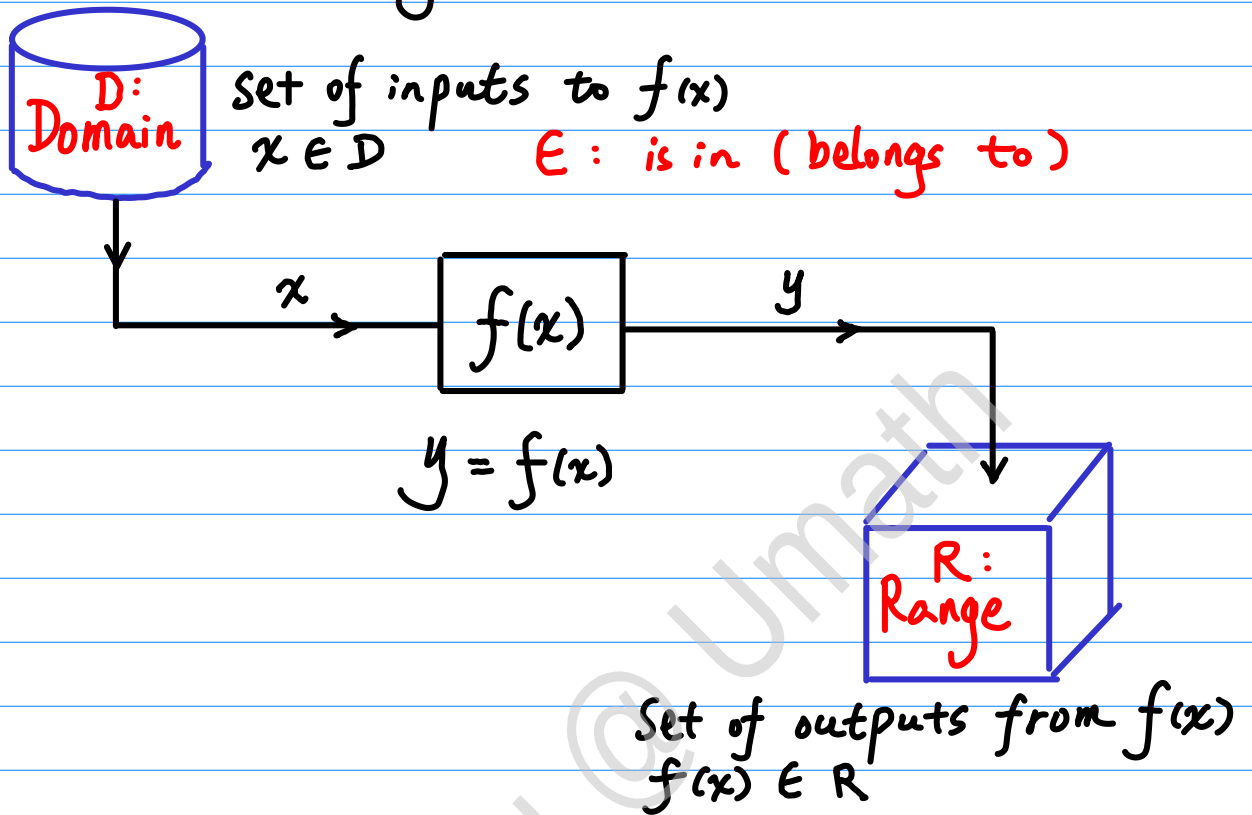
or

$$y = 2x + 3$$

output

rule to produce output
from the input x

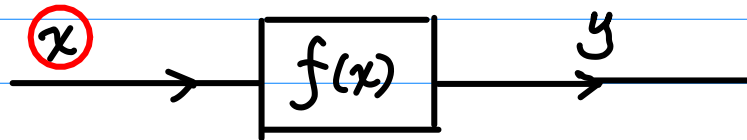
* Domain and Range



Note: If the domain of the function $y = f(x)$ is not specified, then the implied domain of the function f is the set of all allowed inputs.

Evaluating Functions and Solving Function Equations

* Evaluation of Functions



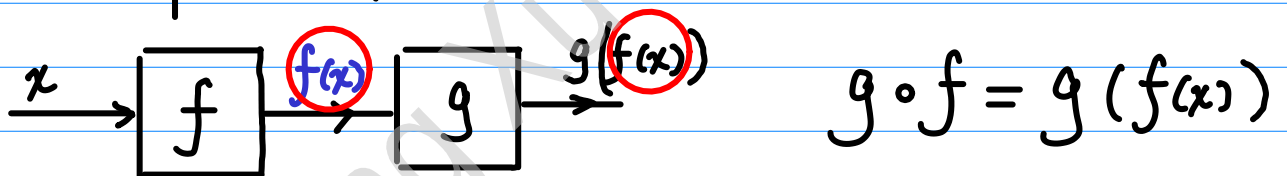
Note: Treat x as a place holder!

$f(\bigcirc)$

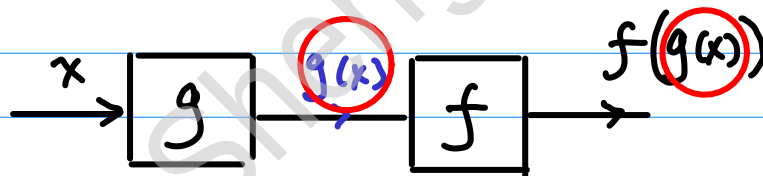
The text 'f' followed by a red circle. A red arrow points upwards from the bottom of the circle.

number or expression;
substitution of x in the expression for $f(x)$
by whatever enclosed by $\bigcirc$

* Composite functions



$$g \circ f = g(f(x))$$

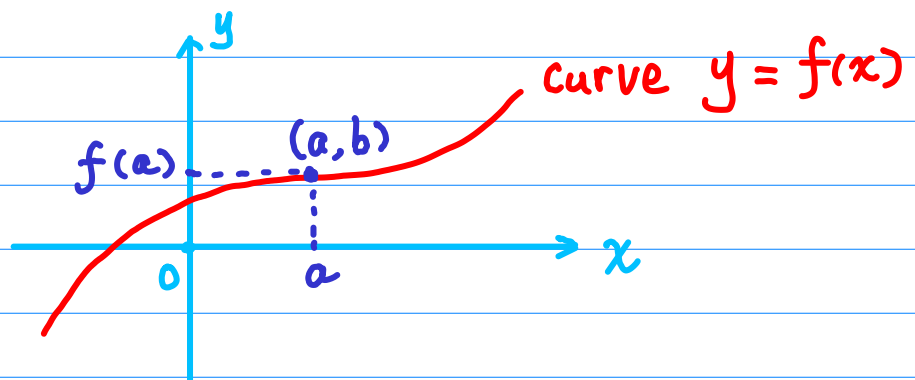


$$f \circ g = f(g(x))$$

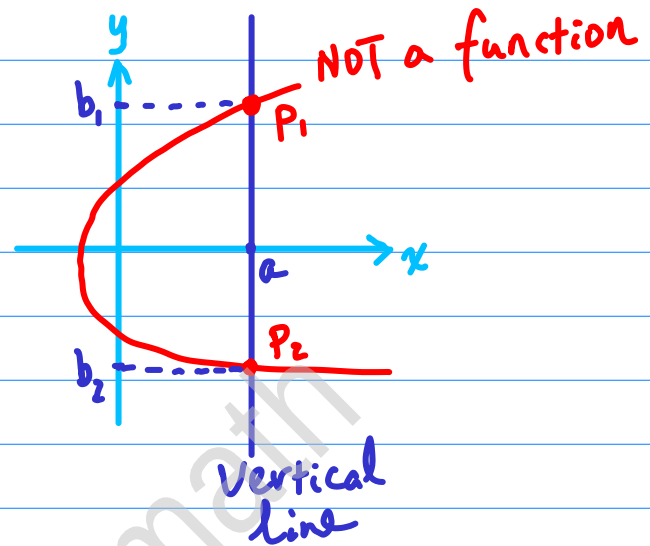
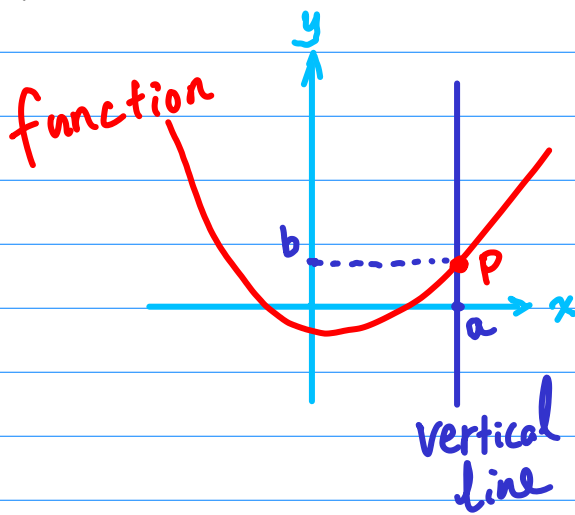
* Function Equations:

Try to substitute various values or expressions for the variables in the function equation with given conditions and needed solutions in mind.

* Cartesian Coordinate System
(rectangular) ↑ the y-axis



- Vertical line test

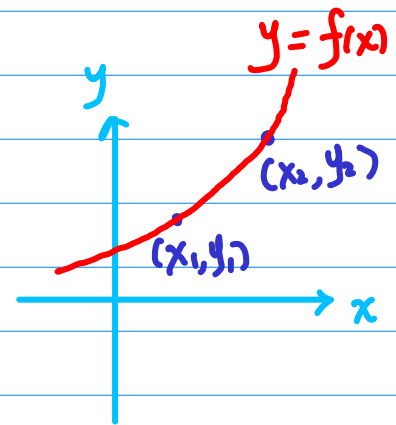


A function: ONE output assigned to each input

- Sketch the graph

Points by Table :

x	$y = f(x)$
x_1	y_1
x_2	y_2
$\vdots$	$\vdots$

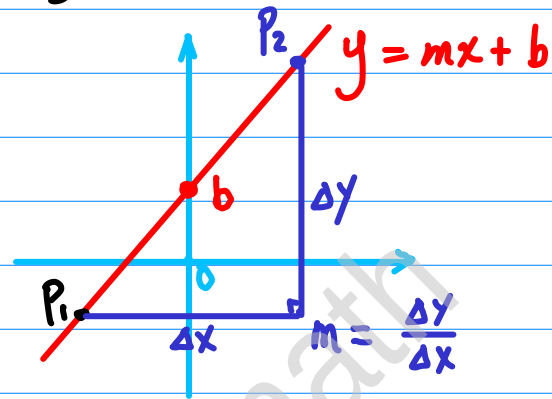


Analytical approach : algebra (& calculus)

Linear Functions and Lines

* slope - intercept form of a line

$$y = mx + b$$

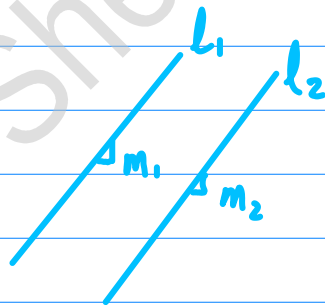


- $P_1(x_1, y_1), P_2(x_2, y_2)$ on the line

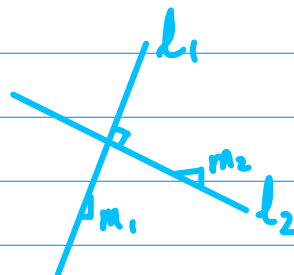
$$\text{slope } m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept b : $(0, b)$ = intersection of line and y-axis

- parallel and perpendicular lines



$$l_1 \parallel l_2 \iff m_1 = m_2$$



$$l_1 \perp l_2 \iff m_1 m_2 = -1$$

* Other forms of a line

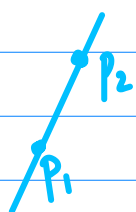
- point-slope form : $y - y_1 = m(x - x_1)$



where $m = \text{slope}$

$(x_1, y_1) = \text{a point on the line}$

- point-point form : $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$



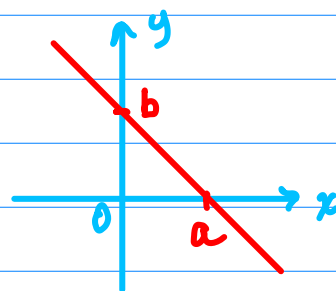
where $(x_1, y_1), (x_2, y_2)$: two points on the line

$m : \text{slope}$

- intercept form : $\frac{x}{a} + \frac{y}{b} = 1$

$x\text{-intercept} = (a, 0)$

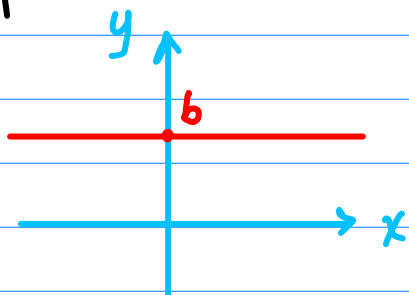
$y\text{-intercept} = (0, b)$



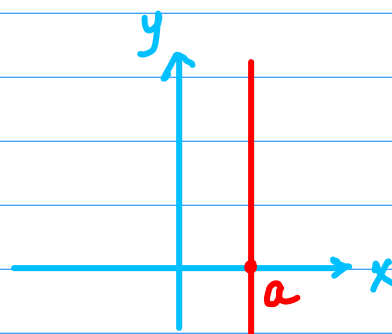
- standard form:

$$Ax + By + C = 0$$

- special lines



$y = b$: slope $m = 0$
Constant function
 $f(x) = b$



$x = a$: slope undefined
NOT graph of a function