

Algebra 2 : Do-you-need Test

1/ Computation (without calculators)

$$(1) \quad z = 1+i : \frac{z \bar{z}}{|z| - i} = ?$$

$$(2) \quad \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)^{20} = ?$$

$$(3) \quad \sqrt{3 - 2\sqrt{2}} = a + b\sqrt{2} : a = ? , b = ?$$

$$(4) \quad e^{2\ln 3} = ?$$

$$(5) \quad \frac{1}{\log_2 6} + \frac{1}{\log_3 6} = ?$$

$$(6) \quad 1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \dots = ?$$

2/ Functions

$$(1) \quad f(x) = \frac{3}{2+x} : \text{Find its inverse function}$$

$$(2) \quad \text{Sketch the graph of } f(x) = 2|1-x| + 1$$

$$(3) \quad f(x^2) = 3x^2 - 2$$

$$f(f(x-1)) = ?$$

3/ Solving Equations

$$(1) \quad \sqrt{3x-2} - \sqrt{x-1} = 1$$

$$(2) \quad x^3 = i, \text{ where } i = \sqrt{-1}$$

$$(3) \quad \begin{cases} x + y + z = 1 \\ 2x + 2y + 3z = -2 \\ 3x + y - z = 13 \end{cases}$$

$$(4) \quad e^{2x} - 2020 = 0$$

4/ Solving Inequalities

(1) $|2x - 3| \leq 2$

(2) $x^2 - 6x + 5 > 0$

(3) $\frac{x^2 - 1}{x + 2} > 0$

(4) $\log_2(3x) \leq 5$

5/ Polynomials

(1) The sum of the two roots of a quadratic equation is 9, and the product of the two roots is 20. Find the two roots.

(2) Find possible rational roots of

$$6x^3 - 17x^2 + ax - 35 = 0$$

where a is an integer.

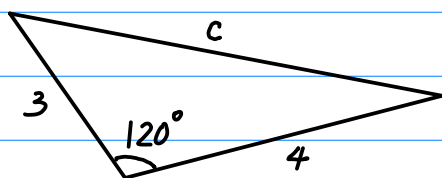
(3) Find the remainder of

$$(9x^4 + x^3 - 12x + 21) \div (x - 1)$$

6/ Analytic Geometry

A line through $(2, 0)$ has a positive slope. If it intersects a circle whose center is $(-1, 0)$ and whose radius is 1 at only one point, find its slope

7/ Trigonometry



(1) Area of the triangle = ?

(2) $c = ?$

8/ Others

- (1) Find the minimum value of $x + \frac{1}{x}$ if $x > 0$.
- (2) Find the coefficient of $x^3 y^4$ in the expansion of $(\frac{x}{2} + zy)^7$.
- (3) Prove that $n! > 2^n$ for $n \geq 4$ using proof by induction
- (4) Prove that $r\pi$ is irrational if r is a nonzero rational number using proof by contradiction.

* 9/ Challenging problems

- (1) $f(x+y) + f(x-y) = 2x^2 + 2y^2$.
 $f(x) = ?$
- (2) $P(x)$ is a 3rd degree polynomial satisfying $P(0) = 0$, $P(1) = \frac{1}{2}$, $P(2) = \frac{2}{3}$ and $P(3) = \frac{3}{4}$.
Find $P(4)$.
- (3) $a > 0$, $b > 0$, $a^{\log_b a} = 2$, $b^{\log_a b} = 16$.
Find a .
- (4) $P_0(x) = x^3 + x^2 + x + 2020$
For $n \geq 1$, define $P_n(x) = P_{n-1}(x-n)$.
What is the coefficient of x in $P_{20}(x)$?