

Congruent Triangles

1

Congruence

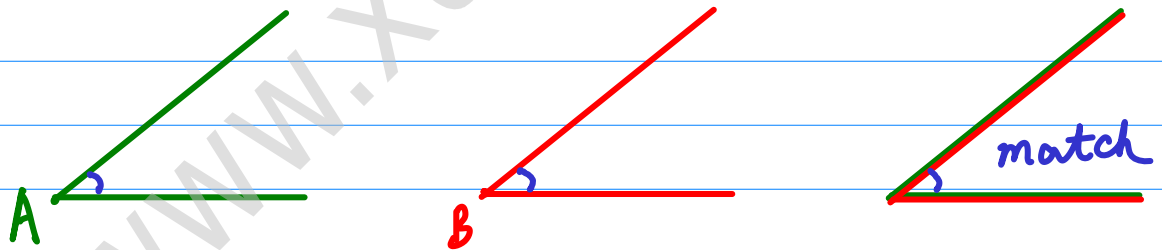
- * Congruent geometric figures :
Same shape and size ; copy of each other

Congruent Segments



$$\overline{AB} \cong \overline{CD} \text{ if and only if } AB = CD$$

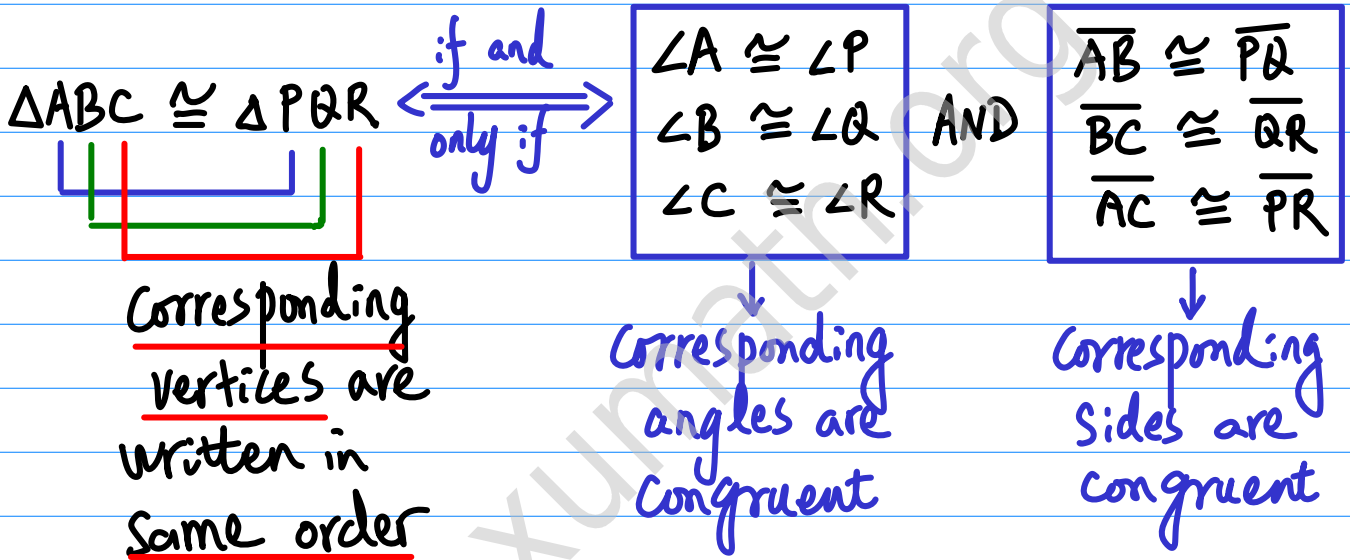
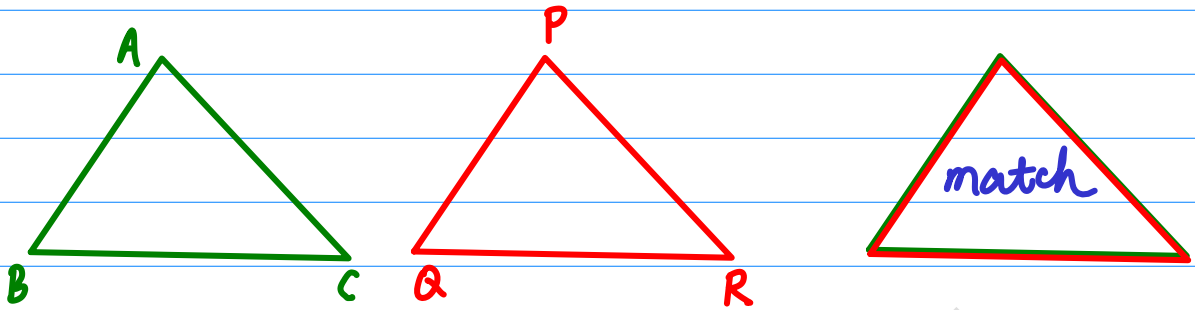
Congruent Angles



$$\angle A \cong \angle B \text{ if and only if } m\angle A = m\angle B$$

Congruent Triangles

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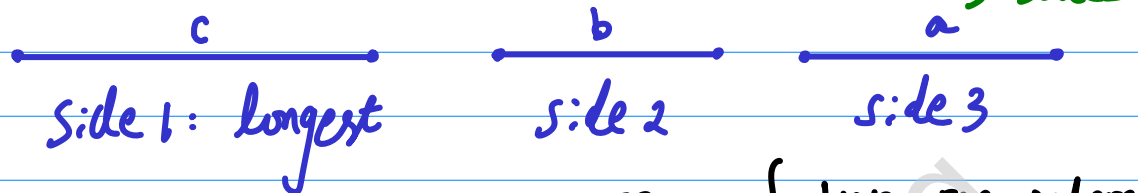


* Note: If congruence of triangles is written correctly, i.e. corresponding vertices are written in the same order, then corresponding angles and sides can be easily identified by reading the writing.

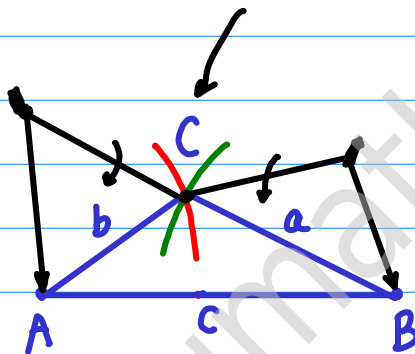
SSS Congruence Postulate

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Construction of a Triangle with Given SSS



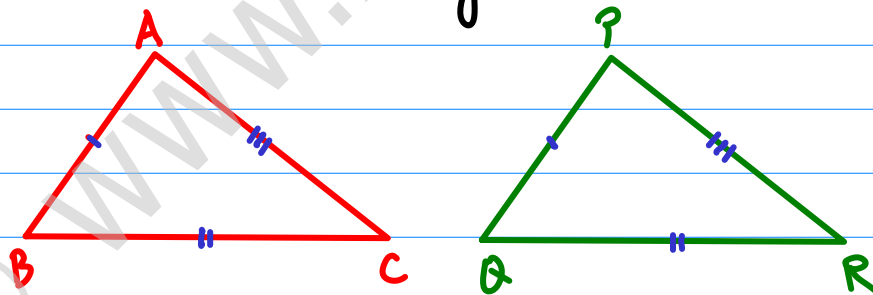
STEP 1: arc with center A and radius b



STEP 3: if two arcs intersect at C, segments $\overline{AC}$, $\overline{BC}$ for $\triangle ABC$

STEP 2: arc with center B and radius a

SSS (side-side-side) Congruence Postulate

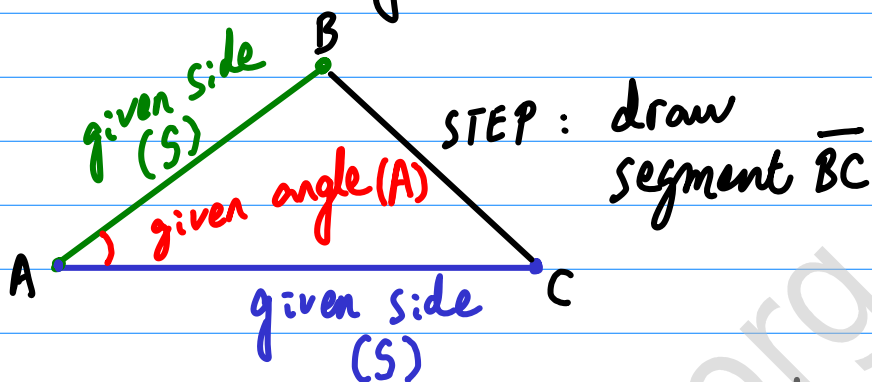


If $\overline{AB} \cong \overline{PQ}$, $\overline{BC} \cong \overline{QR}$, $\overline{AC} \cong \overline{PR}$,
then $\triangle ABC \cong \triangle PQR$

SAS Congruence Postulate

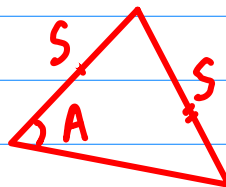
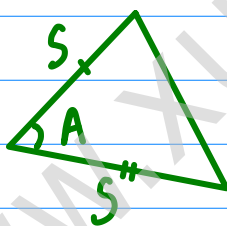
4

Construction of a Triangle with Given SAS



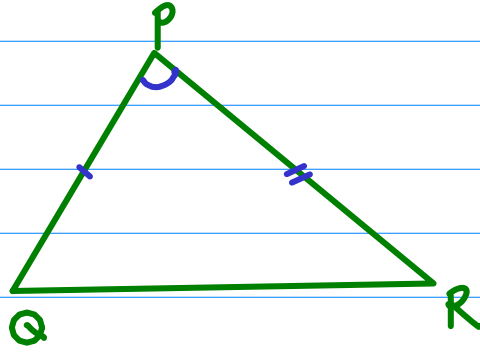
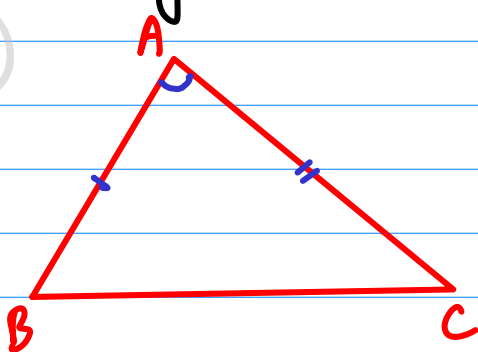
Note: • In SAS, angle A is included between the two sides..

- SAS : different from ASS ☹ or SSA



SAS $\neq$ ASS / SSA

SAS (Side-Angle-Side) Congruence Postulate

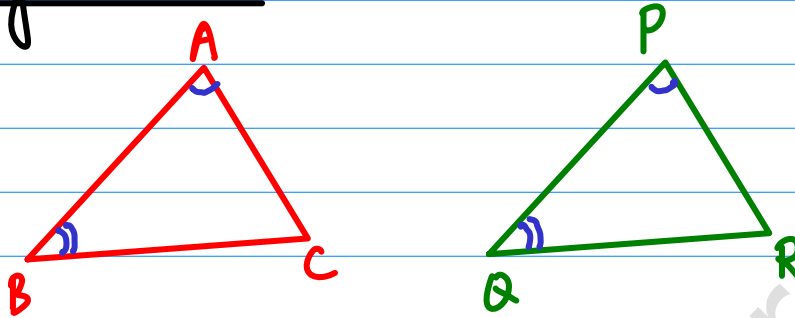


If $\overline{AB} \cong \overline{PQ}$, $\angle A \cong \angle P$, $\overline{AC} \cong \overline{PR}$,
Then $\triangle ABC \cong \triangle PQR$

ASA and AAS Congruence

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Third angle theorem



If $\angle A \cong \angle P$, $\angle B \cong \angle Q$,
Then $\angle C \cong \angle R$

Reason: $m\angle A + m\angle B + m\angle C = 180^\circ$
 $m\angle P + m\angle Q + m\angle R = 180^\circ$
Same $\Rightarrow$ Same

Note: If two angles (AA) of a triangle are congruent to two angles of second triangle, then all three angles (AAA) of one triangle are congruent to all three angles of the other triangle.

i.e.

$AA \Rightarrow AAA$

given 2 angles $\Rightarrow$ given 3 angles

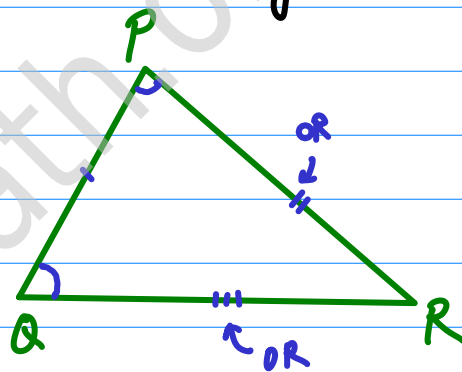
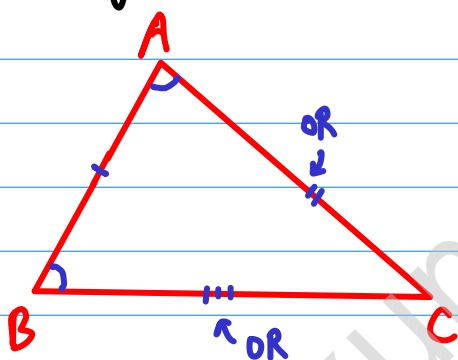
Note: $ASA \text{ or } AAS \Rightarrow AAA + S$

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given 2 angles and 1 side $\Rightarrow$
given 3 angles and 1 side.

ASA and AAS Congruence

("AA+S" congruence i.e. "AAA+S" congruence)



If $\angle A \cong \angle P$, $\angle B \cong \angle Q$ and

$\overline{AB} \cong \overline{PQ}$ (ASA)	$\left. \begin{array}{l} ASA \\ \Downarrow \\ AAS \end{array} \right\}$	$\left. \begin{array}{l} AA+S \\ \Downarrow \\ AAA+S \end{array} \right\}$
or $\overline{AC} \cong \overline{PR}$ (AAS)		
or $\overline{BC} \cong \overline{QR}$ (AAS)		

Then $\triangle ABC \cong \triangle PQR$

* Reason: Construction given ASA ($\Leftrightarrow$ AAS)

