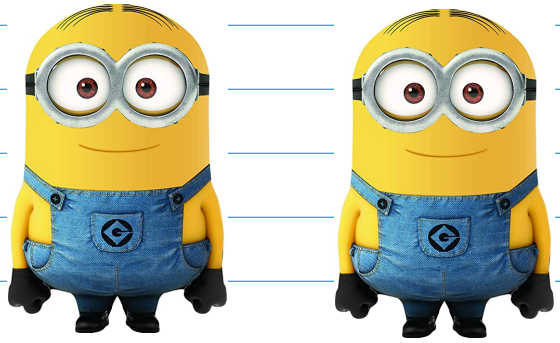
Congruent Triangles

Congruence

* Congruent geometric figures :
  Same shape and size ; copy of each other

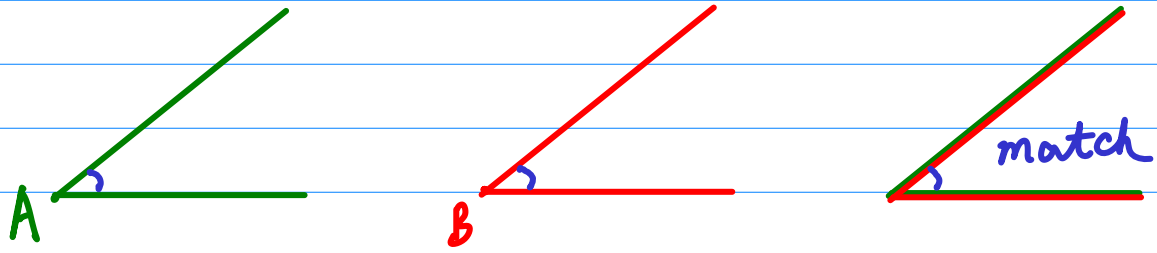


congruent minions : one is the copy of the other

Congruent Segments

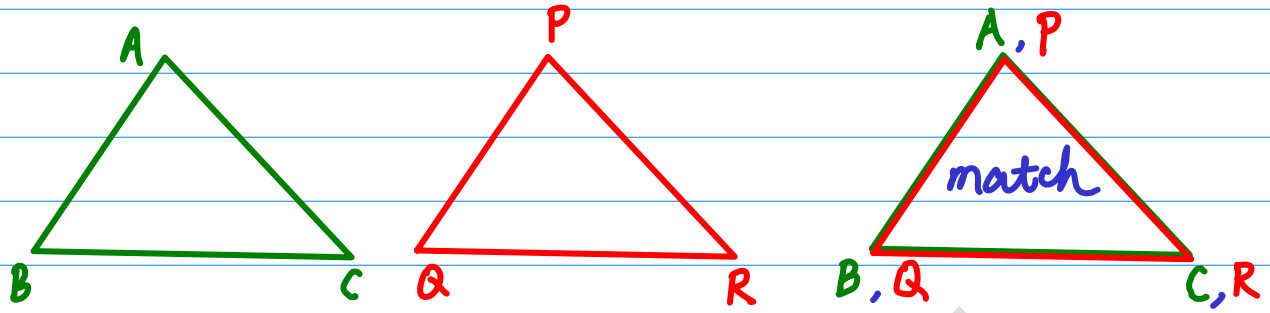
$\overline{AB} \cong \overline{CD}$ if and only if $AB = CD$

Congruent Angles



$\angle A \cong \angle B$ if and only if $m\angle A = m\angle B$

Congruent Triangles



$\Delta ABC \cong \Delta PQR$ if and only if $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$ AND $\overline{AB} \cong \overline{PQ}$, $\overline{BC} \cong \overline{QR}$, $\overline{AC} \cong \overline{PR}$

Corresponding vertices are written in same order

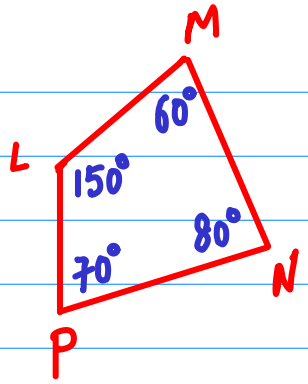
Corresponding angles are congruent

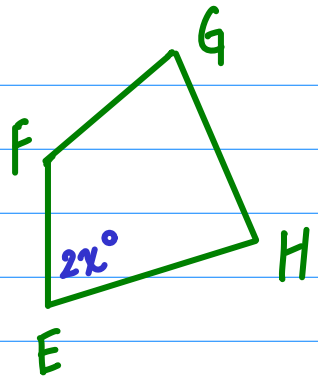
Corresponding sides are congruent

* Note: If congruence of triangles is written correctly, i.e. corresponding vertices are written in the same order, then corresponding angles and sides can be easily identified by reding the writing.

E.g. $\Delta LMN \cong \Delta UVW$: $\begin{cases} \angle M \cong \angle V \\ \overline{LN} \cong \overline{UW} \end{cases}$

poll 1

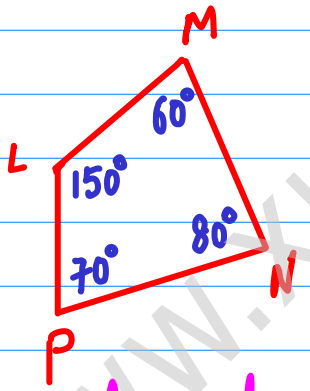


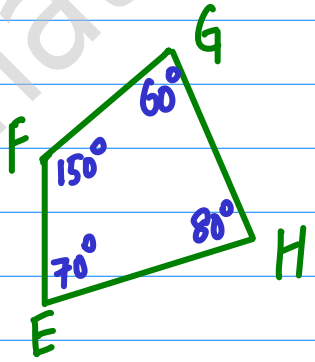


If quadrilaterals (4 sides) LMNP ≅ FGHE, then x =

A. 35° B. 75° C. 30° D. 40°

poll 2





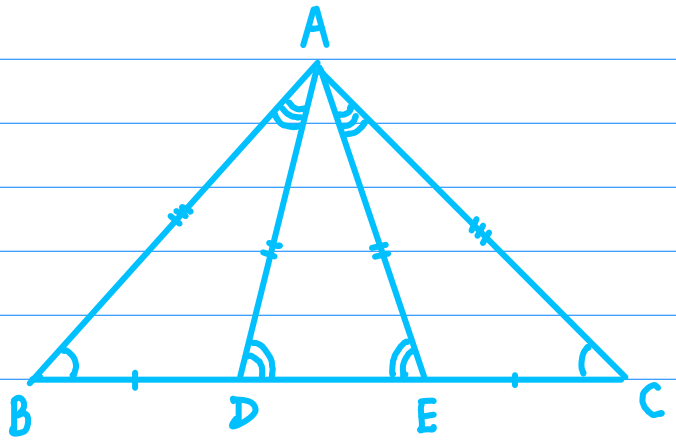
Quadrilaterals LMNP ≅ FGHE,

can you write MNPL ≅ EFGH ?

A. Yes B. No

Note: How about NPLM ≅ HEFG ?

* E.g.



Are ΔABE and ΔADC congruent?

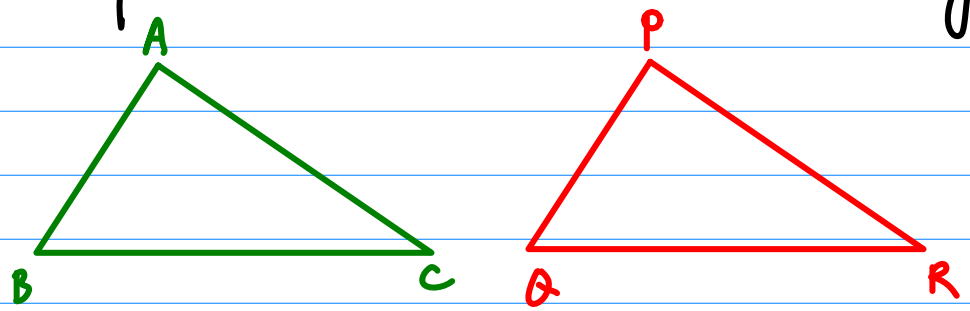
	in ΔABE		in ΔADC	
sides	AB	$=$	AC	
	AE	$=$	AD	
	BE	$=$	CD	(why?)
angles	$m\angle B$	$=$	$m\angle C$	
	$m\angle AEB$	$=$	$m\angle ADC$	
	$m\angle BAE$	$=$	$m\angle DAC$	(why?)

So the two triangles are congruent

we write $\Delta BEA \cong \Delta CDA$

Match vertices of corresponding angles

* How much information is needed to test congruence?



Poll 3

ΔABC and ΔPQR satisfy $\angle A \cong \angle P$ and $\angle B \cong \angle Q$. Then $\angle C$ and $\angle R$:

A. congruent B. noncongruent

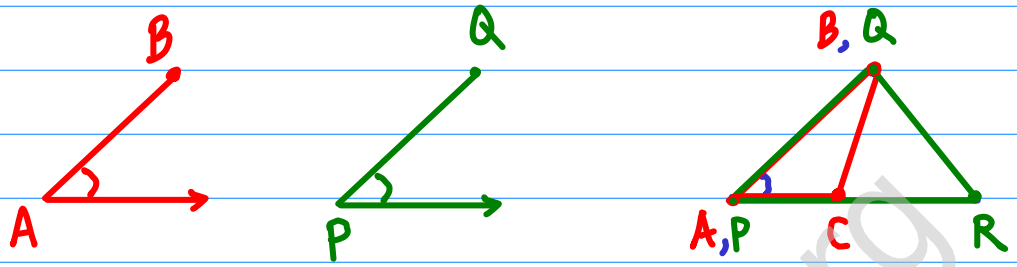
C. Congruent or noncongruent, depending on other information

Note: Third angle theorem.

Note: Do we have to explicitly show congruence of all pairs of corresponding angles and sides to test the congruence? No.

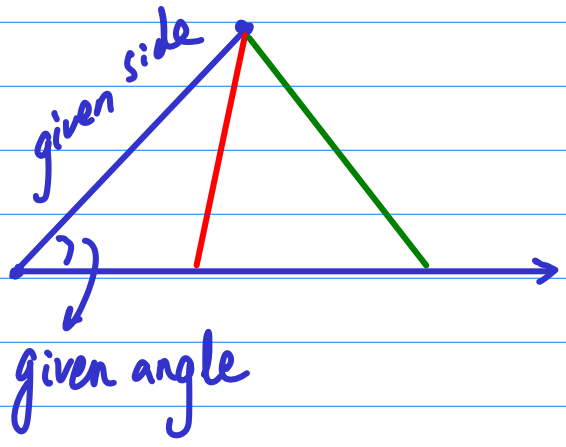
E.g. Consider ΔABC and ΔPQR

If we know only $\angle A \cong \angle P$ and $AB = PQ$:



Then ΔABC can be noncongruent to ΔPQR.

Note: Another way to show ΔABC can be noncongruent to ΔPQR is to construct two noncongruent triangles from one given angle and one given side.

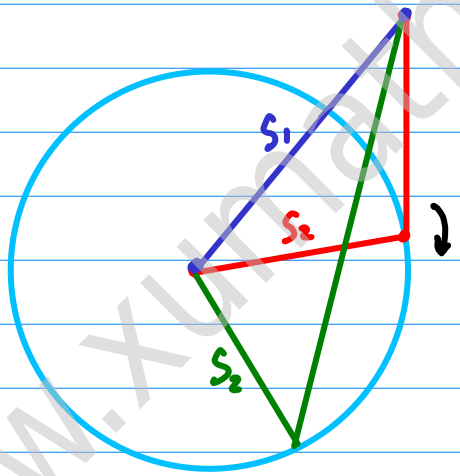


poll 4 Consider ΔABC and ΔPQR. If:

$AB = PQ = s_1$, $AC = PR = s_2$,

can we claim that ΔABC and ΔPQR must be congruent?

A. Yes B. No



SSS Congruence Postulate

Construction of a Triangle with Given SSS

3 sides

c — side 1: longest

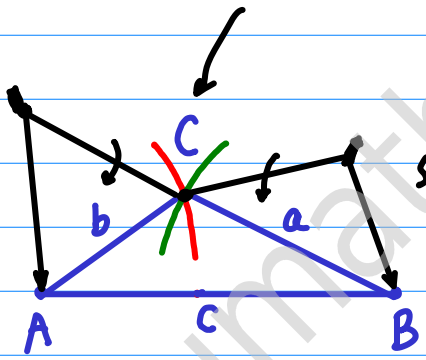
b — side 2

a — side 3

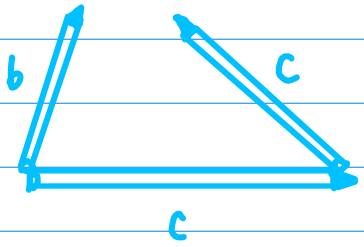
STEP 1: arc with center A and radius b

STEP 2: arc with center B and radius a

STEP 3: if two arcs intersect at C, segments $\overline{AC}$, $\overline{BC}$ for $\triangle ABC$



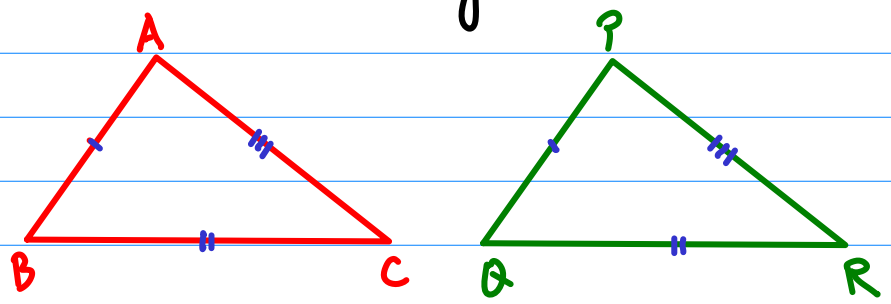
Poll 5 Experiment: Use three pencils to make a triangle. Can you make two noncongruent triangles with different shapes or sizes?



A. Yes B. No.

Note: If one pencil is much longer than the other two, a triangle can NOT be made. (more later)

SSS (side-side-side) Congruence Postulate

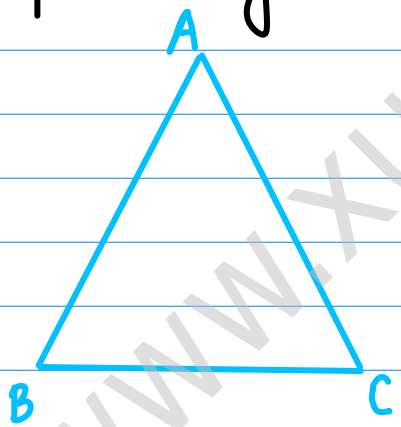


If $\overline{AB} \cong \overline{PQ}$, $\overline{BC} \cong \overline{QR}$, $\overline{AC} \cong \overline{PR}$,

then $\triangle ABC \cong \triangle PQR$

* Examples using SSS congruence

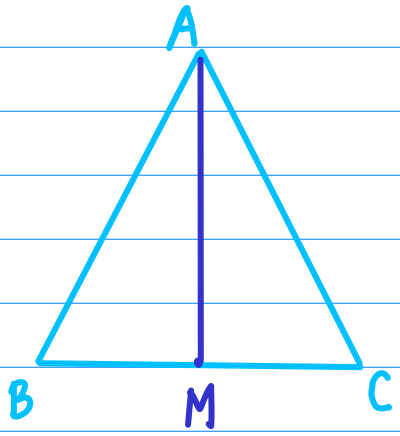
1.



Given: $\triangle ABC$ is an isosceles triangle with base $\overline{BC}$

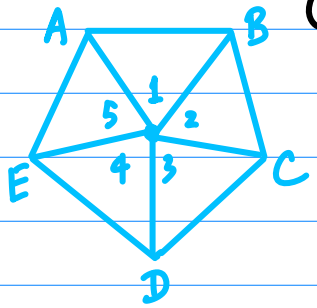
Prove: $m\angle B = m\angle C$

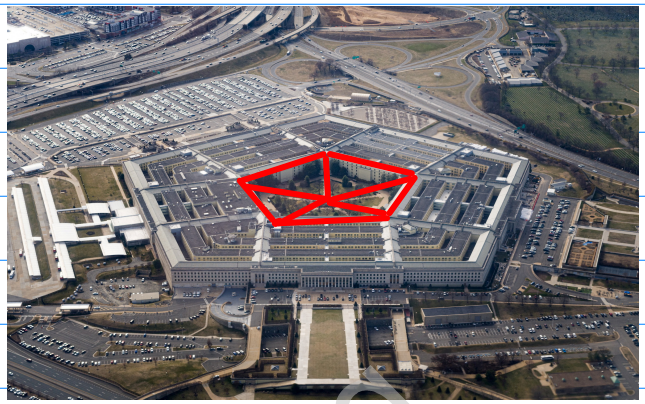
[Proof:] Let M be the midpoint of the base $\overline{BC}$.
Draw segment $\overline{AM}$.



⋮ (in class)

2. The five triangles in the pentagon are isosceles triangles



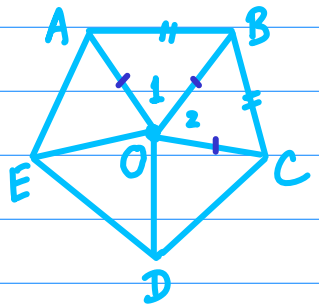


The bases of the five triangles are congruent.

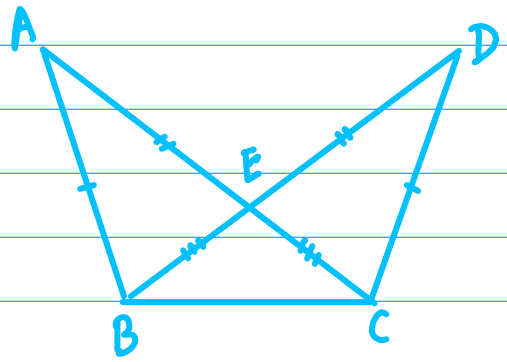
Prove $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = m\angle 5 = 72^\circ$.

[Proof:] Consider $\triangle OAB$ and $\triangle OBC$.

⋮ (in class)



Poll 6

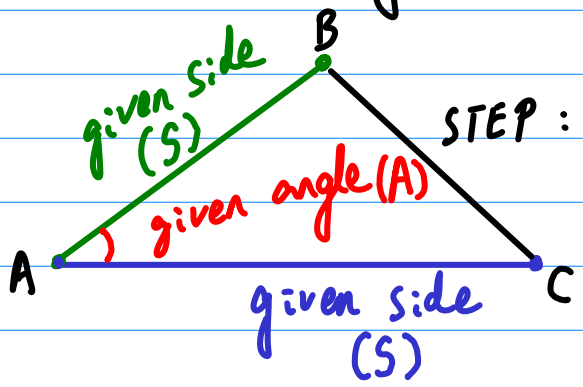


$AB = DC$
$AE = DE$
$BE = CE$
Then ΔABC and ΔDCB

A. congruent B. noncongruent
C. more information is needed to determine if they are congruent

SAS Congruence Postulate

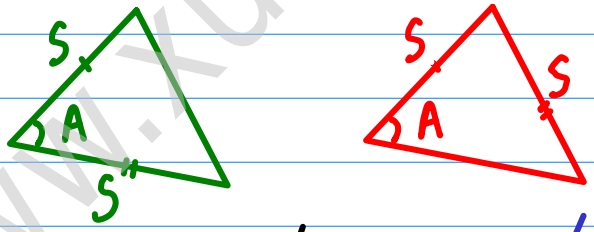
Construction of a Triangle with Given SAS



STEP : draw segment $\overline{BC}$

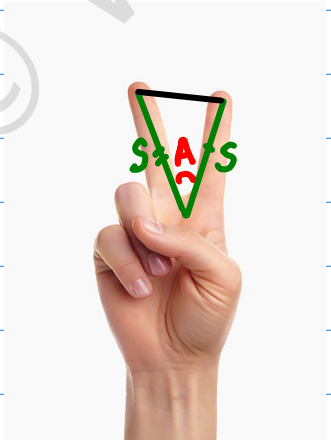
Note: • In SAS, angle A is included between the two sides..

• SAS : different from ASS ☹ or SSA



SAS ≠ ASS / SSA

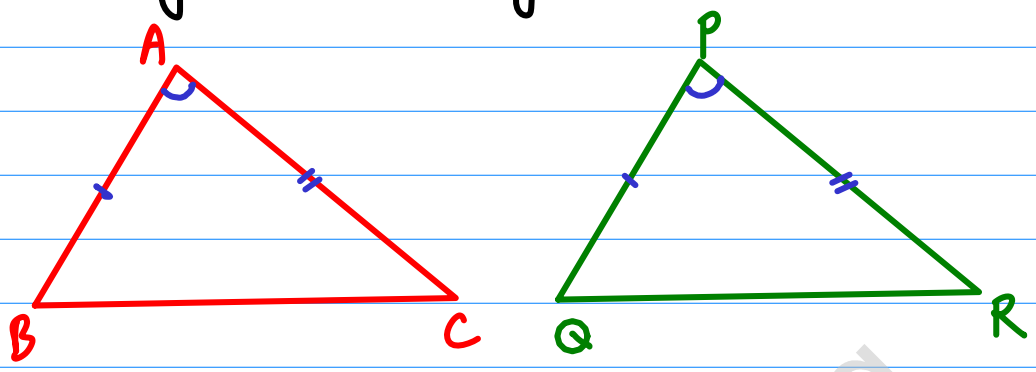
poll 7



Extend two fingers to form a fixed angle. How many triangles can be formed by the two fingers and the segment connecting the two finger tips?

A. 1 B. 2 C. 3

SAS (Side-Angle-Side) Congruence Postulate

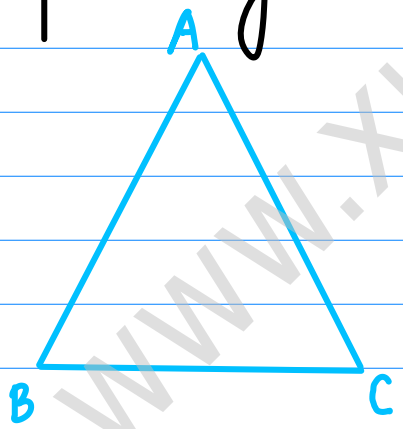


If $\overline{AB} \cong \overline{PQ}$, $\angle A \cong \angle P$, $\overline{AC} \cong \overline{PR}$,

Then $\triangle ABC \cong \triangle PQR$

* Examples using SAS congruence

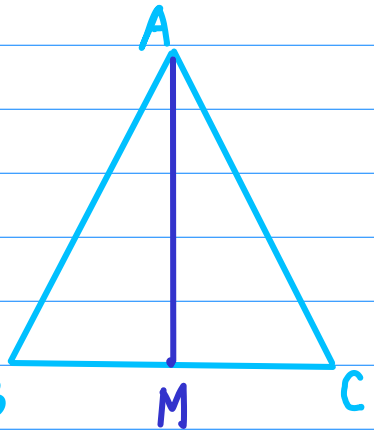
1.



Given: $\triangle ABC$ is an isosceles triangle with base $\overline{BC}$

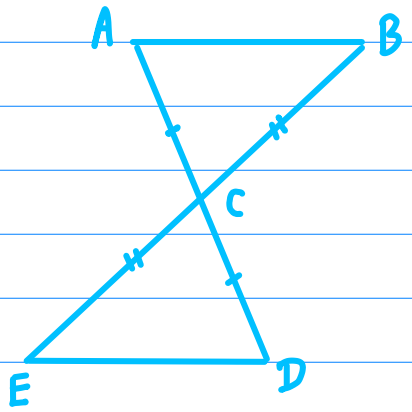
Prove: $m\angle B = m\angle C$

[Proof:] Draw the angle bisector of $\angle A$ to intersect the base $\overline{BC}$ at M.



⋮ (in class)

2.



Given $AC = CD$

$BC = CE$

Prove $AB = DE$

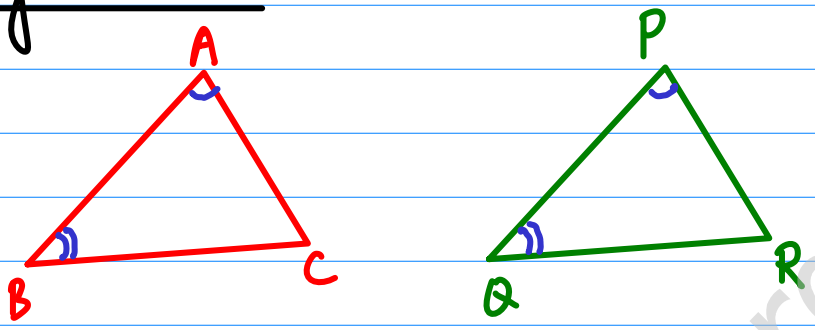
[Proof:] ⋮ (in class)

poll 8 If two legs of a right triangle are congruent to two legs of a second right triangle, then the two right triangles are congruent.

A. True B. False C. Can not tell

ASA and AAS Congruence

Third angle theorem



If $\angle A \cong \angle P$, $\angle B \cong \angle Q$,

Then $\angle C \cong \angle R$

Reason: $m\angle A + m\angle B + m\angle C = 180°$

$m\angle P + m\angle Q + m\angle R = 180°$

Same $\Rightarrow$ same

Note: If two angles (AA) of a triangle are congruent to two angles of second triangle, then all three angles (AAA) of one triangle are congruent to all three angles of the other triangle.

i.e. AA $\Longrightarrow$ AAA

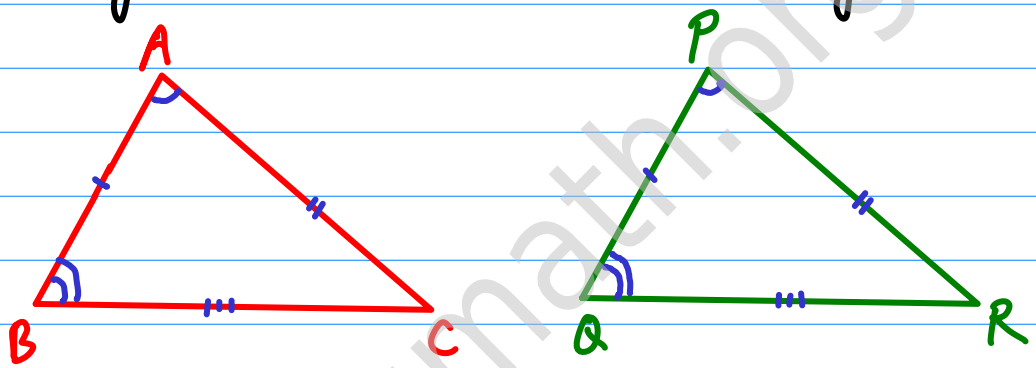
given 2 angles $\Rightarrow$ given 3 angles

Note: **ASA or AAS $\Rightarrow$ AAA + S**

given 2 angles and 1 side $\Rightarrow$ given 3 angles and 1 side.

ASA and AAS Congruence

("AA+S" congruence i.e. "AAA + S" congruence)



If $\angle A \cong \angle P$, $\angle B \cong \angle Q$ and

$\overline{AB} \cong \overline{PQ}$ (ASA)

or $\overline{AC} \cong \overline{PR}$ (A_1A_2S) } AAS

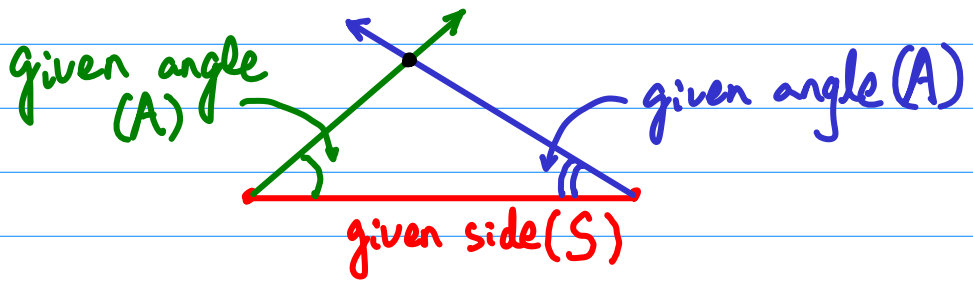
or $\overline{BC} \cong \overline{QR}$ (A_1A_2S) } AAS

AA+S $\Leftrightarrow$ AAA+S

Then $\Delta ABC \cong \Delta PQR$

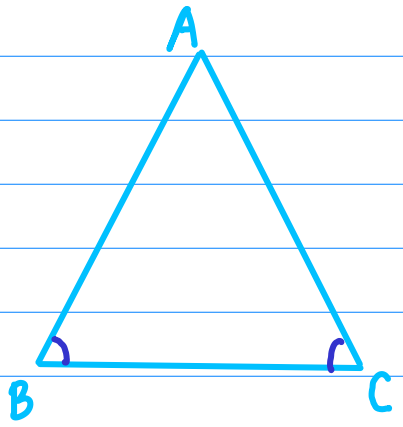
Note: Pay attention to where S is relative to A_1 and A_2

* Reason: Construction given ASA



* Examples using ASA/AAS congruence

1.

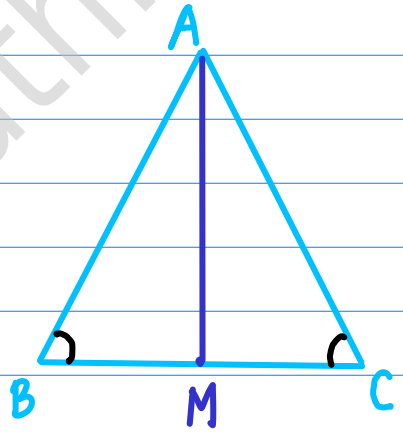


Given: $m\angle B = m\angle C$

Prove: $AB = AC$

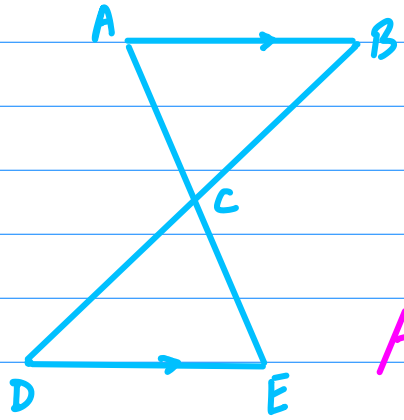
i.e. $\triangle ABC$ is isosceles.

[Proof:] Draw the angle bisector of $\angle A$ to intersect the base $\overline{BC}$ at M.



⋮ (in class)

poll 9



$\overline{AB} \parallel \overline{DE}$

$AC = EC$

Then $\triangle ABC \cong \triangle DEC$?

A. Yes B. No. C. Maybe

Summary

Congruence of triangles :

- SSS
- SAS
- ASA or AAS

(note: AA+S $\Longrightarrow$ AAA + S)